Department of Mathematics & Philosophy of Engineering

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Course: MPZ 3231-Engineering Mathematics IA **Model Answer No·02**

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1)

a. i. $(1-\cos 2y)dx + 2x\sin 2y\ \ dy = 0$

$\frac{\partial F}{\partial x}dx + \frac{\partial F}{\partial y}\ \ dy = 0$ is form of the exact equation;

$\frac{\partial F}{\partial x} = 1-\cos 2y --- (1)$

$\frac{\partial F}{\partial y} = 2x\sin 2y --- (2)$

$\frac{\partial}{\partial y}\left(\frac{\partial F}{\partial x}\right) = 2\sin 2y$ $\quad\quad$ $\frac{\partial}{\partial x}\left(\frac{\partial F}{\partial y}\right) = 2\sin 2y$

$\frac{\partial^2 F}{\partial y\partial x} = 2\sin 2y$ $\quad\quad$ $\frac{\partial^2 F}{\partial x\partial y} = 2\sin 2y$

$\therefore \frac{\partial^2 F}{\partial y\partial x} = \frac{\partial^2 F}{\partial x\partial y} = 2\sin 2y$

$\therefore$ The equation is exact

$\frac{\partial F}{\partial x} = 1-\cos 2y$

$F = \int (1-\cos 2y)\,dx + c(y)$

$\quad = x - x\cos 2y + c(y)$

$\frac{\partial F}{\partial y} = 2x\sin 2y + c'(y)$

By comparing (2)

$c'(y) = 0$

$c'(y) = k$

$F \ \ = x - x\cos 2y + k = 0$

$\therefore$ solution of the equation is

$x - x\cos 2y = k_1$

ii. $(3x^2+2y+1)dx + (2x^2+6y^2+2)\mathrm{dy} = 0$

$\frac{\partial F}{\partial x} = 3x^2+2y+1 --- (1)$ $\quad\quad$ $\frac{\partial F}{\partial y} = 2x+6y^2+2 --- (2)$

$\frac{\partial^2 F}{\partial y\partial x} = 2$ $\quad\quad$ $\frac{\partial^2 F}{\partial x\partial y} = 2$

$\therefore$ the equation is exact.

$$\frac{\partial F}{\partial x} = 3x^2 + 2y + 1$$
$$F = \int (3x^2 + 2y + 1)\,\partial x + c(y)$$
$$F = x^3 + 2xy + x + c(y)$$
$$\frac{\partial F}{\partial y} = 2x + c'(y)$$
By comparing with (2)
$$2x + c'(y) = 2x + 6y^2 + 2$$
$$c'(y) = 6y^2 + 2$$
$$c(y) = \int (6y^2 + 2)dy + d$$
$$= 2y^3 + 2y + d$$

$$F = x^3 + 2xy + x + 2y^3 + 2y + d$$
Solution is $x^3 + 2xy + x + 2y^3 + 2y = d$

b. i.

$$2x^3\frac{dy}{dx} = y^2 + 3xy^2$$

$y = 1$ when $x = 1$

$$2x^3\frac{dy}{dx} = y^2(1 + 3x)$$
$$\int \frac{1}{y^2}dy = \int \frac{(1+3x)}{2x^3}dx + c$$
$$\frac{1}{y} = \left(\frac{1}{2}\times -\frac{1}{2}\times\frac{1}{x^2}\right) - \left(\frac{3}{2}\times\frac{1}{x}\right) + c$$

$$\frac{1}{y} = \frac{1}{4x^2} + \frac{3}{2x} + c$$
$y = 1$ when $x = 1$

$$1 = \frac{1}{4} + \frac{3}{2} + c \qquad c = 1 - \frac{7}{4} = -\frac{3}{4}$$
$$\frac{1}{y} = \frac{1}{4x^2} + \frac{3}{2x} - \frac{3}{4}$$

$$\frac{1}{y} = \frac{1}{4x^2} + \left(\frac{6-3x}{4x}\right)$$
$$\frac{1}{y} = \frac{1 + 6x - 3x^2}{4x^2}$$
$$4x^2 = y(1 + 6x - 3x^2)$$

ii.

$$e^x \frac{dy}{dx} + xy^2 = 0 \qquad y = \frac{1}{2} \text{ when } x \to \infty$$

$$\int y^{-2}dy + \int xe^{-x}dx + c = 0$$

$$\int xe^{-x}dx = -xe^{-x} + \int e^{-x}dx = -xe^{-x} - e^{-x} = -e^{-x}(x+1)$$

$$\int y^{-2}dy = -\frac{1}{y}$$

$$-\frac{1}{y} - e^{-x}(x+1) + c = 0$$

when $x \to \infty$ $e^{-x} = 0$ $\therefore e^{-x}(x+1) = 0$

$$-2 + c = 0$$

$$c = 2$$

$$-\frac{1}{y} - e^{-x}(x+1) + 2 = 0$$

$$y = \frac{1}{2 - e^{-x}(x+1)}$$

c.

$$y + x(x+1)\frac{dy}{dx} = x(x+1)^2 e^{-x^2}$$

$$\frac{dy}{dx} + \frac{y}{x(x+1)} = (x+1)e^{-x^2} \quad --- (1)$$

$$p(x) = \frac{1}{x(x+1)} = \frac{1}{x} - \frac{1}{(x+1)}$$

$$\int p(x)\, dx = \ln x - \ln(x+1) = \ln\frac{x}{x+1}$$

$$e^{\int p(x)dx} = \frac{x}{x+1}$$

$$(1) \times \frac{x}{x+1}$$

$$\frac{x}{x+1}\frac{dy}{dx} + \frac{y}{(x+1)^2} = xe^{-x^2}$$

$$\frac{d}{dx}\left(\frac{x}{x+1}y\right) = xe^{-x^2}$$

$$\frac{x}{x+1}y = \int xe^{-x^2}\, dx + c$$

$$= -\frac{1}{2}\int e^{-x^2}\, d(x^2) + c$$

$$= -\frac{1}{2}e^{-x^2} + c$$

$$xy = (x+1)\left(-\frac{1}{2}e^{-x^2} + c\right)$$

ii

$$x(x+1)\frac{dy}{dx} - y = 3x^4$$

$$\frac{dy}{dx} - y\left(\frac{-1}{x(x+1)}\right) = \frac{3x^4}{x(x+1)}$$

$$\frac{dy}{dx} + y\left(\frac{-1}{x(x+1)}\right) = \frac{3x^3}{(x+1)} --- (1)$$

$$p(x) = \frac{-1}{x(x+1)} = \frac{1}{(x+1)} - \frac{1}{x}$$

$$\int p(x) = \ln\frac{|x+1|}{x}$$

$$e^{\int p(x)} = \frac{x+1}{x}$$

$$(1) \times \frac{x+1}{x}$$

$$\left(\frac{x+1}{x}\right)\frac{dy}{dx} - y\left(\frac{-1}{x(x+1)}\right)\left(\frac{x+1}{x}\right) = \frac{3x^3}{(x+1)}\left(\frac{x+1}{x}\right)$$

$$\left(\frac{x+1}{x}\right)\frac{dy}{dx} - \frac{y}{x^2} = 3x^2$$

$$\frac{d}{dx}\left(\frac{x+1}{x}y\right) = 3x^2$$

$$\left(\frac{x+1}{x}\right)y = \int 3x^2\,dx$$

$$\left(\frac{x+1}{x}\right)y = x^3 + c$$

$$(x+1)y = x^4 + cx$$

d.

i.

$$\frac{dy}{dx} = \frac{x^2 + xy + y^2}{x^2}$$

Since both the numerator and denominator of the right side are of the second degree.

∴ the given equation is homogeneous.

$$\frac{dy}{dx} = 1 + \frac{y}{x} + \left(\frac{y}{x}\right)^2$$

Let $\frac{y}{x} = v$

$$y = vx$$

$$\frac{dy}{dx} = x\frac{dv}{dx} + v$$

$$\frac{dy}{dx} = 1 + v + v^2$$

$$x\frac{dv}{dx} + v = 1 + v + v^2$$
$$\int \frac{dv}{1+v^2} = \int \frac{1}{x}\,dx + c$$
$$\tan^{-1} v = \ln x + c$$
$$\tan^{-1}\frac{y}{x} = \ln x + c$$
$$\tan^{-1}\frac{y}{x} = \ln xc$$
$$xc = e^{\tan^{-1}\frac{y}{x}}$$

ii.

$$\frac{dy}{dx} = \frac{2y - x + 5}{2x - y - 4}$$

Let $y = Y + b$

$x = X + a$

$$\frac{dy}{dx} = \frac{dY}{dX}$$

$$\frac{dY}{dX} = \frac{2Y + 2b - X - a + 5}{2X + 2a - Y - b - 4}$$

$2b - a + 5 = 0$

$2a - b - 4 = 0$

$4a - 2b - 8 = 0$

$3a - 3 = 0$

$a = 1$

$b = -2$

$$\frac{dY}{dX} = \frac{2Y - X}{2X - Y} = \frac{2\,{}^{Y}\!/_{X} - 1}{2 - {}^{Y}\!/_{X}}$$

$$\frac{Y}{X} = V$$
$$\frac{dY}{dX} = X\frac{dV}{dX} + V$$

$$X\frac{dV}{dX} + V = \frac{2V - 1}{2 - V}$$
$$X\frac{dV}{dX} = \frac{2V - 1 - V(2 - V)}{2 - V} = \frac{V - 1}{2 - V}$$
$$\int \frac{2 - V}{V - 1}\,dV = \int \frac{1}{X}\,dX + c$$
$$-\int \frac{V - 1 - 1}{V - 1}\,dV = \int \frac{1}{X}\,dX + c$$
$$-\int \left(1 - \frac{1}{V - 1}\right) dV = \int \frac{1}{X}\,dX + c$$

$$V - \ln|V - 1| = \ln X + c$$
$$V - c = \ln \frac{X}{V - 1}$$

$$\frac{Y}{X} - c = \ln \frac{X}{\frac{Y}{X} - 1}$$
$$\frac{Y - cX}{x} = \ln \frac{X^2}{Y - x}$$

$$\frac{y - 2 - c(x + 1)}{x + 1} = \ln \frac{(x + 1)^2}{y - 2 - (x + 1)}$$
$$\frac{y - 2 - cx - c}{x + 1} = \ln \left[\frac{(x + 1)^2}{y - x - 3}\right]$$

e.

i. $(2x - y + 2)dx + (4x - 2y - 1)dy = 0$

$$[(2x - y) + 2] + [2(2x - y) - 1]\frac{dy}{dx} = 0$$
$$2x - y = t$$
$$2 - \frac{dy}{dx} = \frac{dt}{dx}$$
$$\frac{dy}{dx} = 2 - \frac{dt}{dx}$$
$$(t + 2) + (2t - 1)\left(2 - \frac{dt}{dx}\right) = 0$$
$$(t + 2 + 4t - 2) - (2t - 1)\frac{dt}{dx} = 0$$
$$5t - (2t - 1)\frac{dt}{dx} = 0$$
$$\int \frac{2t - 1}{5t}\, dt = \int dx + c$$
$$\frac{2}{5}t - \frac{1}{5}\ln t = x + c$$

$$2(2x - y) - \ln(2x - y) = 5x + d$$

ii.

$$\frac{dy}{dx} = x^3y^2 + xy - - - (1)$$
$$\frac{(1)}{y^2}$$

$$\frac{1}{y^2}\,\frac{dy}{dx} = x^3 + \frac{x}{y}$$

$$\frac{1}{y^2}\frac{dy}{dx} - \frac{x}{y} = x^3$$

$$t = -\frac{1}{y}$$

$$\frac{dt}{dx} = -\frac{1}{y^2}\frac{dy}{dx}$$

$$\frac{dt}{dx} + xt = x^3$$

$$p(x) = x$$

$$\int p(x)dx = \frac{x^2}{2}$$

I.F. $= e^{\frac{x^2}{2}}$

$$e^{\frac{x^2}{2}}\frac{dt}{dx} + xte^{\frac{x^2}{2}} = x^3e^{\frac{x^2}{2}}$$

$$\frac{d}{dx}\left(te^{\frac{x^2}{2}}\right) = x^3e^{\frac{x^2}{2}}$$

$$te^{\frac{x^2}{2}} = \int x^3e^{\frac{x^2}{2}}dx$$

Let $I = \int x^3e^{\frac{x^2}{2}}dx = \int x^2.xe^{\frac{x^2}{2}}dx$

$$u = x^2$$

$$\frac{dv}{dx} = xe^{\frac{x^2}{2}}$$

$$v = \int xe^{\frac{x^2}{2}}dx$$

$$v = e^{\frac{x^2}{2}}$$

$$I = x^2e^{\frac{x^2}{2}} - \int 2x\, e^{\frac{x^2}{2}}dx$$

$$= x^2e^{\frac{x^2}{2}} - 2\int e^{t'}dt$$

$$t' = \frac{x^2}{2}$$

$$\frac{dt'}{dx} = x$$

$$= x^2e^{\frac{x^2}{2}} - 2e^{\frac{x^2}{2}}$$

$$\therefore te^{\frac{x^2}{2}} = x^2e^{\frac{x^2}{2}} - 2e^{\frac{x^2}{2}} + c$$

$$-\frac{1}{y} = x^2 - 2 + ce^{\frac{x^2}{2}}$$

f.

i.

$$\frac{d^2y}{dx^2}+\frac{dy}{dx}-2y=0$$

Let $y=e^{mx}$

$$\frac{dy}{dx}=me^{mx} \quad \frac{d^2y}{dx^2}=m^2e^{mx}$$

$(m^2+m-2)e^{mx}=0$

$e^{mx}\neq 0 \;\; m^2+m-2=0$

$(m+2)(m-1)=0$

$m=-2 \;\;, \;\; m=1$

General solution is $y=Ae^{-2x}+Be^{x}$

ii.

$$\frac{d^3x}{dt^3}-2\frac{d^2x}{dt^2}-3\frac{dx}{dt}=0$$

Let $x=e^{mt}$

$$\frac{dx}{dt}=me^{mx} \quad \frac{d^2x}{dt^2}=m^2e^{mx} \quad \frac{d^3x}{dt^3}=m^3e^{mx}$$

$(m^3-2m^2-3m)e^{mx}=0$

$e^{mx}\neq 0 \;\; m^3-2m^2-3m=0$

$m(m-3)(m+1)=0$

$m=0, m=3 \;\; \& \;\; m=-1$

General solution is $x=Ae^{0t}+Be^{3t}+Be^{-t}$

$$x=A+Be^{3t}+Be^{-t}$$

2. $f=-kv^4$

$$\frac{dv}{dt} = -kv^4$$

$$\int_u^v \frac{dv}{kv^4} = -\int_0^t dt$$

$$\left[\frac{1}{k}\left(-\frac{1}{3}\right)\frac{1}{v^3}\right]_u^v = [-t]_0^t$$

$$\frac{-1}{3k}\left[\frac{1}{v^3} - \frac{1}{u^3}\right] = -t$$

$$t = \frac{1}{3k}\left[\frac{1}{v^3} - \frac{1}{u^3}\right]$$

$$f = v\frac{dv}{dx}$$

$$\therefore v\frac{dv}{dx} = -kv^4$$

$$\int_u^v \frac{dv}{v^3} = -k\int_0^x dx$$

$$\left[\left(-\frac{1}{2}\right)\frac{1}{v^2}\right]_u^v = k[-x]_0^x$$

$$-\frac{1}{2}\left[\frac{1}{v^2} - \frac{1}{u^2}\right] = -x$$

$$x = \frac{1}{2k}\left[\frac{1}{v^2} - \frac{1}{u^2}\right]$$

3.

a. $27x + 6y - z = 85$

$$6x + 15y + 2z = 72$$

$$x + y + 54z = 110$$

$$x = \frac{1}{27}(85 - 6y + z)$$

$$y = \frac{1}{15}(72 - 6x - 2z)$$

$$z = \frac{1}{54}(110 - x - y)$$

Jacobi method

$$x_1{}^{k+1} = \frac{1}{27}(85 - 6y^k + z^k)$$

$$y_2{}^{k+1} = \frac{1}{15}(72 - 6x^k - 2z^k)$$

$$z_3{}^{k+1} = \frac{1}{54}(110 - x^k - y^k)$$

Starting with $(x_1{}^0, y_1{}^0, z_1{}^0) = (0,0,0)$

$$x_1{}^1 = \frac{1}{27}(85) = 3.15$$

$$y_2{}^1 = \frac{1}{15}(72) = 4.8$$

$$z_3{}^1 = \frac{1}{54}(110) = 2.04$$

$(x^1, y^1, z^1) = (3.15 , 4.8 , 2.04)$

$$x^2 = \frac{1}{27}(85 - 6 \times 4.8 + 2.04) = 2.16$$

$$y^2 = \frac{1}{15}(72 - 6 \times 3.15 - 2 \times 2.04) = 3.27$$

$$z^2 = \frac{1}{54}(110 - 3.15 - 4.8) = 1.89$$

$(x^2, y^2, z^2) = (2.16 , 3.27 , 1.89)$

$$x^3 = \frac{1}{27}(85 - 6 \times 3.27 + 1.89) = 2.49$$

$$y^3 = \frac{1}{15}(72 - 6 \times 2.16 - 2 \times 1.89) = 3.68$$

$$z^3 = \frac{1}{54}(110 - 2.16 - 3.27) = 1.94$$

$(x^3, y^3, z^3) = (2.49 , 3.68 , 1.94)$

$$x^4 = \frac{1}{27}(85 - 6 \times 3.68 + 1.94) = 2.40$$

$$y^4 = \frac{1}{15}(72 - 6 \times 2.49 - 2 \times 1.94) = 3.55$$

$$z^4 = \frac{1}{54}(110 - 2.49 - 3.68) = 1.92$$

Similarly

$(x^5, y^5, z^5) = (2.43\ , 3.58\ , 1.93)$

$(x^6, y^6, z^6) = (2.42\ , 3.57\ , 1.93)$

$(x^7, y^7, z^7) = (2.43\ , 3.57\ , 1.93)$

$x = 2.43, \quad y = 3.57, \quad z = 1.93$

b. $27x + 6y - z = 85$

$6x + 15y + 2z = 72$

$x + y + 54z = 110$

Gauss Seidel Method

$$x^{k+1} = \frac{1}{27}(85 - 6y^k + z^k)$$

$$y^{k+1} = \frac{1}{15}(72 - 6x^{k+1} - 2z^k)$$

$$z^{k+1} = \frac{1}{54}(110 - x^{k+1} - y^{k+1})$$

Starting with $(x^0, y^0, z^0) = (0,0,0)$

$$x^1 = \frac{1}{27}(85 - 6 \times 0 + 0) = 3.15$$

$$y^1 = \frac{1}{15}(72 - 6 \times 3.15 - 2 \times 0) = 3.54$$

$$z^1 = \frac{1}{54}(110 - 3.15 - 3.54) = 1.91$$

$(x^1, y^1, z^1) = (3.15\ , 3.54\ , 1.91)$

$$x^2 = \frac{1}{27}(85 - 6 \times 3.54 + 1.91) = 2.43$$

$$y^2 = \frac{1}{15}(72 - 6 \times 2.43 - 2 \times 1.91) = 3.57$$

$$z^2 = \frac{1}{54}(110 - 2.43 - 3.57) = 1.93$$

$(x^2, y^2, z^2) = (2.43\ , 3.57\ , 1.93)$

$$x^3 = \frac{1}{27}(85 - 6 \times 3.57 + 1.93) = 2.43$$

$$y^3 = \frac{1}{15}(72 - 6 \times 2.43 - 2 \times 1.93) = 3.57$$

$$z^3 = \frac{1}{54}(110 - 2.43 - 3.57) = 1.93$$

$x = 2.43, \quad y = 3.57, \quad z = 1.93$

c.

x	0	$\frac{\pi}{2}$	π
y	0	1	0

$$y(x) = \frac{(x - x_1)(x - x_2)}{(x_0 - x_1)(x_0 - x_2)}(y_0) + \frac{(x - x_0)(x - x_2)}{(x_1 - x_0)(x_1 - x_2)}(y_1) + \frac{(x - x_0)(x - x_1)}{(x_2 - x_0)(x_2 - x_1)}(y_2)$$

$$y(x) = \frac{\left(x - \frac{\pi}{2}\right)(x - \pi)}{\left(0 - \frac{\pi}{2}\right)(0 - \pi)}(0) + \frac{(x - 0)(x - \pi)}{\left(\frac{\pi}{2} - 0\right)\left(\frac{\pi}{2} - \pi\right)}(1) + \frac{(x - 0)\left(x - \frac{\pi}{2}\right)}{(\pi - 0)\left(\pi - \frac{\pi}{2}\right)}(0)$$

$$y(x) = \frac{(x)(x - \pi)}{\left(\frac{\pi}{2}\right)\left(-\frac{\pi}{2}\right)} = \frac{4x(x - \pi)}{-\pi^2}$$

$$y\left(\frac{\pi}{4}\right) = \frac{4 \times \frac{\pi}{4}\left(\frac{\pi}{4} - \pi\right)}{-\pi^2} = \frac{\pi\left(\frac{\pi}{4} - \pi\right)}{-\pi^2} = \frac{3}{4} = 0.75$$

$\therefore \sin\frac{\pi}{4} = 0.75$

4. $f(x) = x^2 - 18 = 0$

Let $x = 4 \qquad f(x) = -2 \qquad < 0$

$x = 5 \qquad f(x) = 7 \qquad > 0$

$$x_0 = \frac{5 + 4}{2} = 4.5$$

$f(x_0) = 4.5^2 - 18 \ > 0$

$$x_1 = \frac{4 + 4.5}{2} = 4.25$$

$$f(x_1) = 4.25^2 - 18 = 0.0625 > 0$$

$$x_2 = \frac{4 + 4.25}{2} = 4.125$$

$$f(x_2) = 4.5^2 - 18 = -0.98438 < 0$$

$$x_3 = \frac{4.125 + 4.25}{2} = 4.1875$$

$$f(x_3) = 4.1875^2 - 18 = -0.46484 < 0$$

$$x_4 = \frac{4.1875 + 4.25}{2} = 4.21875$$

$$f(x_4) = 4.21875^2 - 18 = -0.202148 < 0$$

$$x_5 = \frac{4.21875 + 4.25}{2} = 4.234375$$

$$f(x_5) = 4.234375^2 - 18 = -0.07007 < 0$$

$$x_6 = \frac{4.234375 + 4.25}{2} = 4.2421875$$

$$f(x_6) = 4.2421875^2 - 18 = -0.00385 < 0$$

$$x_7 = \frac{4.2421875 + 4.25}{2} = 4.24609375$$

$$f(x_7) = 4.24609375^2 - 18 = 0.029312134 > 0$$

$$x_8 = \frac{44.2421875 + 4.24609375}{2} = 4.24414063$$

$$f(x_8) = 4.24414063^2 - 18 = 0.01273 > 0$$

$$x_9 = \frac{4.24414063 + 4.24609375}{2} = 4.24316406$$

$$f(x_9) = 4.24316406^2 - 18 = 0.004441 > 0$$

$$x_{10} = \frac{4.24316406 + 4.24609375}{2} = 4.24267578$$

$$x = 4.243$$

b. Using Newton Raphson Method

$f(x) = x^2 - 18 = 0$

$f'(x) = 2x$

$$x_{n+1} = x_n - \frac{{x_n}^2 - 18}{2x_n}$$

We can select $x_0 = 4.5$

$$x_1 = x_0 - \frac{4.5^2 - 18}{2x_0} = 4.25$$

$$x_2 = 4.25 - \frac{4.5^2 - 18}{2x_1} = 4.24264706$$

$$x_3 = 4.24264069 - \frac{4.5^2 - 18}{2x_2} = 4.24264069$$

$$x_4 = 4.24264069 - \frac{4.5^2 - 18}{2x_3} = 4.242640696$$

c. By using Newton's forward difference interpolation formula

$$y_s = y_0 + \Delta y_0 \binom{s}{1} + \Delta^2 y_0 \binom{s}{2} + \Delta^3 y_0 \binom{s}{3} + \dots\dots + \Delta^n y_0 \binom{s}{n}$$

Where

$$\binom{s}{r} = \frac{s(s-1)(s-2)\dots\dots..(s+1-r)}{r!}$$

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
0.1	0.1003				
		0.0508			
0.15	0.1511		0.0008		
		0.0516		0.0002	
0.2	0.2027		0.001		0.0002
		0.0526		0.0004	
0.25	0.2553		0.0014		
		0.054			
0.3	0.3093				

$$y_s = y_0 + \Delta y_1 \binom{s}{1} + \Delta^2 y_2 \binom{s}{2} + \Delta^3 y_3 \binom{s}{3} + \Delta^4 y_4 \binom{s}{4}$$

$$s = \frac{0.13 - 0.10}{0.05} = 0.6$$

$\binom{s}{1} = 0.6$

$\binom{s}{2} = \frac{0.6(-0.4)}{2!} = -0.12$

$\binom{s}{3} = \frac{0.6(-0.4)(-1.4)}{3!} = 0.056$

$\binom{s}{4} = \frac{0.6(-1.4)(-2.4)}{4!} = -0.0336$

$y_s = 0.1003 + (0.0508 \times 0.6) + (0.0008 \times -0.12) + (0.0002 \times 0.056)$

$+(0.0002 \times -0.0336)$

$= 0.1307$

$\tan 0.13 = 0.1307$

By using Newton's backward difference interpolation formula

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
0.1	0.1003				
		0.0508			
0.15	0.1511		0.0008		
		0.0516		0.0002	
0.2	0.2027		0.001		0.0002
		0.0526		0.0004	
0.25	0.2553		0.0014		
		0.054			
0.3	0.3093				

$y_s = y_5 - \Delta y_4 \binom{s}{1} + \Delta^2 y_3 \binom{s}{2} - \Delta^3 y_2 \binom{s}{3} + \Delta^4 y_1 \binom{s}{4}$

$s = \frac{(x_7 - x)}{h} = \frac{130 - 1.28}{0.05} = 0.4$

$\binom{s}{1} = 0.4$

$\binom{s}{2} = \frac{0.4(-0.6)}{2!} = -0.12$

$\binom{s}{3} = \frac{0.4(-0.6)(-1.6)}{3!} = 0.056$

$\binom{s}{4} = \frac{0.4(-0.6)(-1.6)(-2.6)}{4!} = -0.0416$

$y_s = 0.3093 + (0.054 \times -0.6) + (0.0014 \times -0.12) + (0.0004 \times -0.056)$

$$+(0.0002 \times -0.0336)$$

$$= 0.276703$$

$\tan 0.27 = 0.276703$

d.

x	0.0	0.2	0.4	0.6	0.8
y	1	0.9430	0.8825	0.7458	0.6855

By using Trapezoidal rule

$$\int_a^b y\,dx = \frac{h}{2}\left[(y_0 + y_n) + 2(y_1 + y_2 + \cdots + y_{n-1})\right]$$

$$\int_0^{0.8} y\,dx = \frac{0.2}{2}\left[(1 + 0.6855) + 2(0.9430 + 0.8825 + 0.7458)\right]$$

$$\int_0^{0.8} y\,dx = 0.68281$$

By using Simpson rule

$$\int_a^b y\,dx = \frac{h}{3}\left[(y_0 + y_{2n}) + 4(y_1 + y_3 + \cdots + y_{2n-1}) + 2(y_2 + y_4 + \cdots + y_{2n-2})\right]$$

$$\int_0^{0.8} y\,dx = \frac{0.2}{3}\left[(1 + 0.6855) + 4(0.9430 + 0.7458) + 2(0.8825)\right]$$

$$\int_0^{0.8} y\,dx = 0.68038$$